CHAPTER 05

Projectile, Circular and Periodic Motion

5.1 MOTION IN TWO DIMENSIONS

The previous chapters have considered motion mainly in a straight line. This is called **recti-linear** motion (Latin *rectus* = 'straight' and *linea* = 'line'). This chapter will be looking at motion in two dimensions, that is, **curvilinear** motion.

Projectiles from cannons, a shotput, throwing a cricket ball, motorcyclists jumping rows of cars; and ballet dancers all involve curvilinear motion.

But there are facts and fallacies about such motion:

- Before Galileo, universities taught that when a cannon ball ran out of 'impetus' it would stop in its path and fall vertically to Earth. That's not true, is it?
- Soldiers in war have often reported that enemy bullets fired from miles away fell vertically in to their trenches. How can that be true?
- In the Olympic Hammer Throw, the hammer continues in a circular path for a fraction of a second after it is let go. True or false?
- Bombs and bullets fired at 45° have the greatest range. Well, cricket balls do; so should bullets.
- A pendulum will swing forever in a vacuum because air resistance is nil. True or false?

To make sense of these ideas, it helps if you have first-hand knowledge of some curvilinear motions.

NEI Activity 5.1 THINGS THAT DON'T GO IN STRAIGHT LINES

1 Watch a microwave oven in operation.

(a) Does the carousel rotate clockwise or anticlockwise? Does everyone else in the class get the same result?

(b) Measure the 'period' of the carousel. This is the time for one complete revolution. Time the carousel for five turns to get better accuracy. Is 12 seconds about the class average?

2 If you have a **CD player** and still have the manual, look up the rotation speed of the disk. Is it constant or is there a range of speeds?

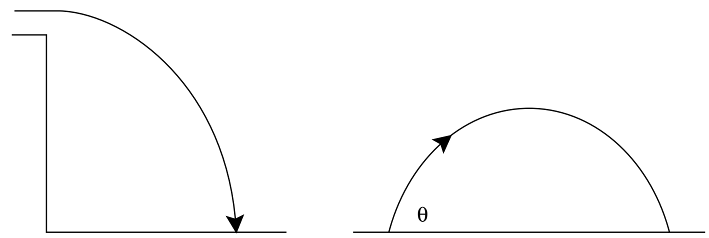
3 **Billiard** players talk about putting 'English' on the ball. What does that mean?

4 The **javelin** design was changed in 1998 so that it couldn't be thrown as far. Consult the *Guinness Book of Records* to find out how this was achieved and by approximately how much its range was reduced.

PROJECTILES 5.2

Good examples of projectiles are 1. a rock thrown straight out from the top of a cliff; 2. a cricket ball thrown across a field. (See Figure 5.1.) The word **projectile** comes from the Latin *jacere* meaning 'to throw' and *pro* meaning 'forward'. Projectile motion can be separated into two components — a **vertical** (up and down) motion and a **horizontal** motion. The vertical motion is the same as discussed in Chapter 2 — the ball is under the influence of gravity and accelerates at -10 m s^{-2} directed downward (the negative direction). In the horizontal direction, there are no net forces acting on the object so the velocity is constant. In all cases we are assuming air resistance is negligible. If you are to ever take air resistance into account in a problem you will be specifically told to do so. The path of a moving object is called its **trajectory** (Latin *trajectus* = 'crossing' or 'passage').

Figure 5.1



Note: in all examples that follow, the positive direction is upward and the negative direction is downward. You may choose a different convention in your problem-solving. It's up to you and your teacher.

Horizontal projection

This is the example of the rock thrown off the cliff. In this case the value of $\boldsymbol{v}_h$ equals the initial horizontal velocity ($\boldsymbol{u}_h$), which remains constant. The vertical velocity starts at zero ($\boldsymbol{u}_v = 0$) but increases as time passes.

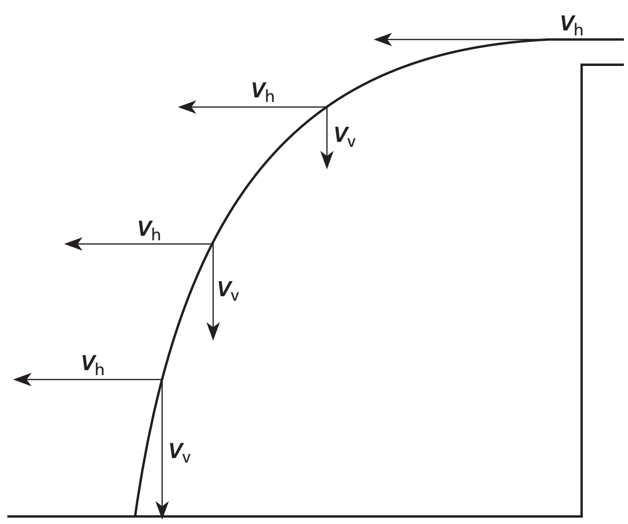


Figure 5.2
The horizontal velocity remains constant while the vertical velocity increases.

Example

A golf ball is thrown horizontally off a cliff at a velocity of 20 m s^{-1} and takes 4 s to reach the ground below. Calculate **(a)** the height of the cliff; **(b)** how far the ball will land from the base of the cliff; **(c)** the impact velocity of the ball.

Solution

(a) In the vertical direction:

$$\boldsymbol{u}_v = 0 \text{ m s}^{-1},\ \boldsymbol{a} = -10 \text{ m s}^{-2},\ t = 4 \text{ s},\ \boldsymbol{s}_v = ?$$

$$\begin{aligned} \boldsymbol{s}_v &= \boldsymbol{u}_v t + \tfrac{1}{2}\boldsymbol{a}t^2 \\ &= 0 + \tfrac{1}{2} \times -10 \times 4^2 \\ &= -80 \text{ m} \end{aligned}$$

(b) In the horizontal direction:

$$\boldsymbol{u}_h = 20 \text{ m s}^{-1},\ \boldsymbol{a} = 0 \text{ m s}^{-2},\ t = 4 \text{ s},\ \boldsymbol{s}_h = ?$$

$$\begin{aligned} \boldsymbol{s}_h &= \boldsymbol{u}_h t + \tfrac{1}{2}\boldsymbol{a}t^2 \\ &= 20 \times 4 + 0 \\ &= 80 \text{ m} \end{aligned}$$

(c) Impact velocity is the sum of horizontal velocity, which remains constant at 20 m s^{-1}, and the final vertical velocity. This is a vector summation. The vertical velocity on impact, $\boldsymbol{v}_v = \boldsymbol{u}_v + \boldsymbol{a}t = 0 + -10 \times 4 = -40$ m s^{-1}.

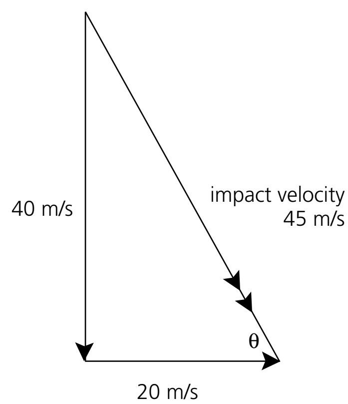


Figure 5.3

Using Pythagoras's theorem:

$$\begin{aligned} \boldsymbol{v}^2 &= 40^2 + 20^2 = 1600 + 400 \\ \boldsymbol{v} &= \sqrt{2000} = 45 \text{ m s}^{-1} \end{aligned}$$

The angle of impact, θ, can be found from $\tan\theta = \frac{40}{20} = 2.0$.

Hence θ = 63°.

Questions

1 A motorcycle is driven off a cliff at a horizontal velocity of 25 m s^{-1} and takes 5 s to reach the ground below. Calculate **(a)** the height of the cliff; **(b)** the distance out from the base of the cliff that the motorcycle lands; **(c)** the impact velocity.

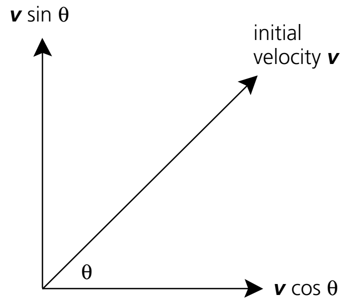
2 A rock is thrown horizontally at 8 m s^{-1} off a 100 m high cliff. Calculate **(a)** how long it takes to hit the ground; **(b)** its impact velocity; **(c)** how far out from the cliff it lands.

PROJECTION AT AN ANGLE 5.3

Not all objects are thrown in a horizontal direction. Cannonballs, footballs and netballs, for example, are often projected upward at an angle.

To study projectile motion, we let θ be the angle at which the object is thrown relative to the horizontal. This is called the **elevation angle**.

Figure 5.4



NOVEL CHALLENGE

A flea can jump 18.4 cm high when jumping at 45°. How far horizontally will it go?

The motion of the projectile is a **parabola** because the vertical displacement varies as a function of t^2 (i.e. $\boldsymbol{s}_v = \boldsymbol{u}_v t + \frac{1}{2}\boldsymbol{a}t^2$) as it is uniformly accelerated motion whereas the horizontal displacement varies with just t (i.e. $\boldsymbol{s}_h = \boldsymbol{v}_h t$) as it is constant velocity. The horizontal displacement is called the **range**.

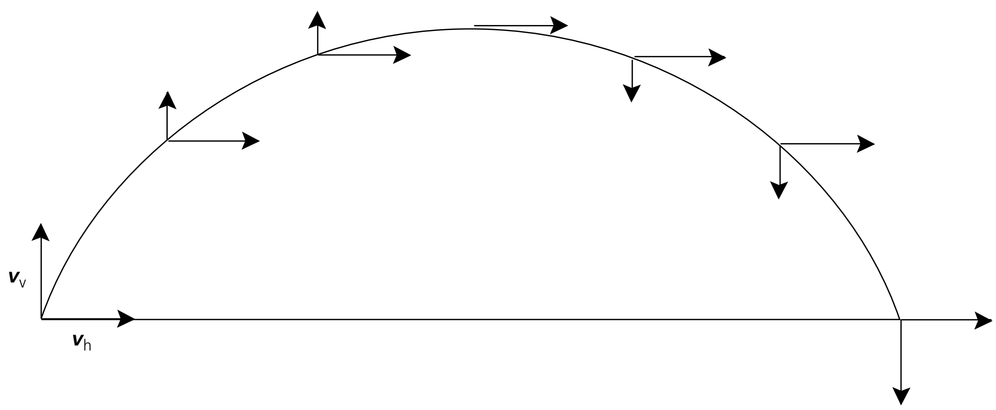
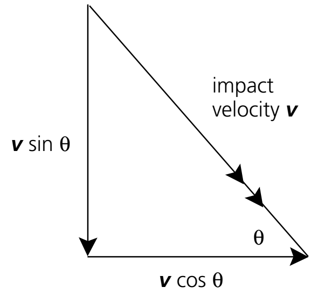


Figure 5.5 The vertical velocity changes while the horizontal velocity stays constant.

The **impact** velocity will have the same magnitude as the **launch** velocity, but be directed in a general downward direction not upward as at launch (Figure 5.6).

Figure 5.6



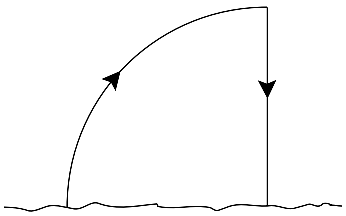
The horizontal component of velocity remains constant for the duration of the flight. The vertical component at launch equals the vertical component at impact but in the opposite direction. Recall from an earlier chapter that for vertical motion, initial speed equals final speed for an object returning to the same horizontal level.

Some old ideas challenged

Until the time of Galileo, the motion of a projectile was based on the teachings of Greek philosopher Aristotle. For example, Albert of Saxony (1316–90), rector of Paris University, taught that the trajectory of a projectile was in three parts: firstly, the upward motion where the initial impetus suppressed gravity; secondly, a period where the projectile's impetus and gravity were compounded; and thirdly, when gravity and air resistance overcame the natural impetus. This produced a trajectory as shown in Figure 5.7.

It wasn't until 1638 that the trajectory of a projectile could be described mathematically. Galileo's description proved to be correct and has been the basis of mechanics since. The mathematical techniques that Galileo pioneered, later refined by Newton, can be seen in the examples that follow.

Figure 5.7
Until the 1600s, people thought that projectile motion was more like this.



Example

The L16 mortar is a weapon currently used by Commonwealth defence forces. If a mortar shell was fired at 200 m s^{-1} at an angle of 40° to the ground, calculate:

(a) the initial vertical and horizontal components of the velocity;
(b) the maximum height reached;
(c) the time of flight (total time taken from start to finish);
(d) the horizontal range;
(e) the impact velocity.

Solution

Let the upward direction be positive: $\boldsymbol{a} = -10$ m s^{-2}.

(a) • Vertical: $\boldsymbol{u}_v = \boldsymbol{v} \sin \theta = 200 \times \sin 40° = +129$ m s^{-1} in positive direction (up).
• Horizontal: $\boldsymbol{u}_h = \boldsymbol{v} \cos \theta = 200 \times \cos 40° = 153$ m s^{-1}.

(b) At maximum height $\boldsymbol{v}_v = 0$ m s^{-1}.

$$(\boldsymbol{v}_v)^2 = (\boldsymbol{u}_v)^2 + 2\boldsymbol{a}\boldsymbol{s}_v, \text{ hence } \boldsymbol{s}_v = \frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{2\boldsymbol{a}} = \frac{0^2 - (+129)^2}{2 \times -10} = +832 \text{ m}$$

(c) Time of flight can either be calculated by **(i)** determining the time taken to reach maximum height ($\boldsymbol{v} = 0$) and doubling it; or **(ii)** determining time taken until final vertical velocity is equal and opposite to initial vertical velocity; or **(iii)** until vertical displacement is zero again.

By **(i)** $\boldsymbol{v}_v = \boldsymbol{u}_v + \boldsymbol{a}t$, hence $t = \frac{\boldsymbol{v} - \boldsymbol{u}}{\boldsymbol{a}} = \frac{0 - (+129)}{-10} = 12.9$ seconds. Total time = 25.8 s.

By **(ii)** $\boldsymbol{v}_v = \boldsymbol{u}_v + \boldsymbol{a}t$, hence $t = \frac{\boldsymbol{v} - \boldsymbol{u}}{\boldsymbol{a}} = \frac{-129 - (+129)}{-10} = 25.8$ s.

By **(iii)** $\boldsymbol{s}_v = \boldsymbol{u}_v t + \frac{1}{2}\boldsymbol{a}t^2$, hence $0 = +129t + -5t^2$; $5t = 129$; hence $t = 25.8$ s.

(d) Horizontal range = horizontal component of initial velocity × time of flight.

$$\boldsymbol{s}_h = \boldsymbol{v}_h \times t = 153 \times 25.8 = 3947 \text{ m}$$

(e) The impact velocity will have the same magnitude as the initial velocity, but will be directed generally downward not up. The angle of impact (θ) will be the same as the angle of elevation (40°). Thus, the impact velocity is 200 m s^{-1} at an angle 40° to the horizontal.

NOVEL CHALLENGE

Acapulco cliff divers jump off a cliff 35 m high and just miss rocks 5 mm out from the base. What is their minimum push-off speed?

NOVEL CHALLENGE

On the Moon, astronauts hit a golf ball 180 m. If they hit the same ball on Earth with the same speed and angle, how far will it go (neglect air resistance)? Note $\boldsymbol{g}_{moon} = 1.6$ m s^{-2}. By the way, there are three golf balls still on the Moon. Learn this off by heart — it could be useful.

Complementary angles of elevation

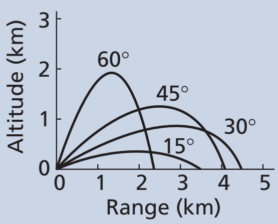
The range of a projectile fired at an elevation angle of 40° will also be the same if it is fired at 50°. The angles 40° and 50° are called **complementary angles** because they add up to 90°. Other examples of complementary pairs are: 30° and 60°; 20° and 70° etc. In other words, the range of a projectile will be the same for elevation angles of θ and 90° – θ. It is interesting that $\sin \theta = \cos (90° - \theta)$.

Activity 5.2 TOY CANNON

If you have access to a toy cannon, try firing some projectiles at complementary angles and collect some data. Perhaps you could design a device that uses a rubber band, a mousetrap or a spring to fire small objects up an incline. Then you could vary the elevation angle. Whatever you do, you should aim to confirm or refute the above assertion about complementary angles.

NOVEL CHALLENGE

The following graphs show how the range and altitude of a projectile changes with elevation angle **in the presence of air**. Plot a graph of maximum altitude versus elevation angle and predict maximum altitude for an angle of 90°. *Should the graph pass through the origin (0,0)? Why?*



Example

In the earlier example, an elevation angle of 40° produced a range of 3947 m. If the theory is correct, then an angle of 50° should produce the same range.

(a) Prove this assertion.
(b) By how much do the times of flight differ?
(c) Do the impact velocities differ? (The initial velocity was 200 m s^{-1}.)

Solution

(a) Let $\boldsymbol{a} = -10$ m s^{-2}.

$$u_v = v \sin \theta = 200 \times \sin 50° = +153 \text{ m s}^{-1} \text{ (upward)}$$
$$u_h = v \cos \theta = 200 \times \cos 50° = 129 \text{ m s}^{-1}$$

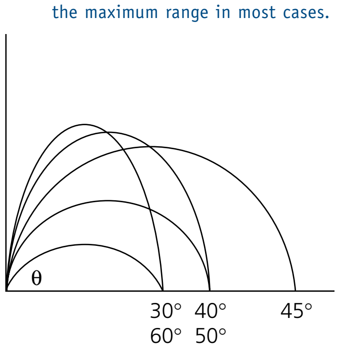
Impact velocity in vertical direction (v_v) = $-u_v$ = -153 m s^{-1}.
Hence, the range is identical.

$$v_v = u_v + at, \text{ hence } t = (v_v - u_v/a) = (-153 - +153)/10 = 30.6 \text{ s}$$
$$s_h = v_h \times t = 129 \times 30.6 = 3947 \text{ m}$$

(b) The times of flight were: for 40°, t = 25.8 s; for 50°, t = 30.6 s; difference was 4.8 s.
(c) Impact velocities are different but only in direction not magnitude.
For 40°, v_{impact} = 200 m s^{-1} at 40° to horizontal.
For 50°, v_{impact} = 200 m s^{-1} at 50° to horizontal.

Maximum range

Figure 5.8
An elevation angle of 45° produces the maximum range in most cases.



It was the invention of the cannon in the late 1400s that created a new form of warfare. War at sea using cannons became more common and defence using medieval castles became obsolete. Medieval mechanics also became obsolete. Until then, the motion of a projectile was only of philosophical interest because they all thought they knew how projectiles moved — after all, Aristotle described the motion over 1000 years earlier and no one was prepared to challenge his theories. The theories weren't challenged until they had to be tested in warfare and were found wanting. Aiming was very much a hit-or-miss affair; there was no way of determining the trajectory or even the angle of launch in advance. It wasn't until self-taught engineer Niccolo Fontana published the results of his experiments in 1546 that gunners realised a 45° angle of elevation would give the maximum range.

In Figure 5.8 the maximum range can be calculated by letting θ = 45°. In this case the horizontal and vertical components of the initial velocity both equal 141 m s^{-1}, the time of flight equals 28.2 s and the maximum range works out to be 3976 m.

SR Activity 5.3 COMPUTER SIMULATION

If you have access to a computer and spreadsheet you may like to use this exhaustive method of determining the range at different elevations and the elevation for maximum range.

1 The horizontal range (R) can be found by a single formula deduced in the following manner:
   (a) Horizontal range $\boldsymbol{s}_h = \boldsymbol{u} \cos\theta \times t = R$; maximum vertical height $\boldsymbol{v}_v = \boldsymbol{u} \sin\theta + \boldsymbol{a}t/2$.
   (b) Eliminating t between the equations yields: $R = \dfrac{2\boldsymbol{u}^2}{\boldsymbol{a}} \sin\theta \cos\theta$.
   (c) Knowing the identity $\sin 2\theta = 2 \sin\theta \cos\theta$, we obtain $R = \dfrac{\boldsymbol{u}^2}{\boldsymbol{a}} \sin 2\theta$.

2 Set up a spreadsheet and calculate the range for all values of θ from 0° to 90° using a nominal velocity of 100 m s^{-1}.

3 Is the maximum range achieved at an elevation of 45°?

4 Do complementary angles produce the same range? Give an example.

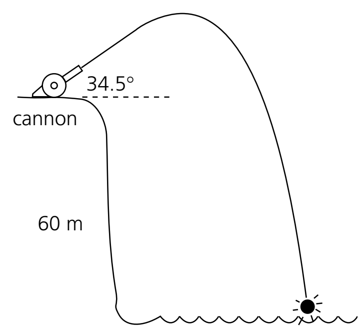
NOVEL CHALLENGE

The world speed record for an archery shot over 100 m is 1.64 seconds (220 km h^{-1}). Calculate the elevation angle of the arrow so that it hits the bull's eye at the same height as that from which it was fired (shoulder high).

More complex situations

If the projectile travels to a point *lower* than its starting point then the situation is more complex. Imagine throwing a ball up and out off a cliff. Another complex situation arises when the projectile lands *higher* up than the starting point, for example throwing a book to someone up on a verandah or shooting a basketball into the hoop.

Figure 5.9(a)



Example: Lower final horizontal displacement

A cannon is fired from the edge of a cliff, which is 60.0 m above the sea (Figure 5.9(**a**)). The cannonball's initial velocity is 88.3 m s^{-1} and it is fired at an upward angle of 34.5° to the horizontal. Determine: **(a)** the time the ball is in the air; **(b)** the impact velocity; **(c)** the horizontal distance out from the base of the cliff that the ball strikes the water.

Solution

- Vertical component of initial velocity $\boldsymbol{u}_v = 88.3 \sin\theta = +50.0$ m s^{-1} (positive is up).
- Horizontal component of initial velocity $\boldsymbol{u}_h = 88.3 \cos\theta = 72.8$ m s^{-1}.
- The final vertical displacement $\boldsymbol{s}_v = -60.0$ m
- Initial vertical velocity $\boldsymbol{u}_v = +50.0$ m.
- Vertical acceleration $\boldsymbol{a} = -10$ m s^{-2}.

$$\boldsymbol{s} = \boldsymbol{u}t + \tfrac{1}{2}\boldsymbol{a}t^2$$
$$-60 = +50t + \tfrac{1}{2}(-10)t^2$$
$$5t^2 - 50t - 60 = 0$$
$$t^2 - 10t - 12 = 0$$
$$t = \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \times 1 \times -12}}{2 \times 1}$$
$$t = 11.1 \text{ s or } -1.1 \text{ s}$$

The negative solution is not reasonable, therefore the time of flight is 11.1 s.

(b) The horizontal velocity $\boldsymbol{v}_h$ remains constant at 72.8 m s^{-1}.

NOVEL CHALLENGE

A really hard one! A cannonball is fired and, after travelling 5 m horizontally, it has reached half its maximum height.

At what horizontal distance will it land?

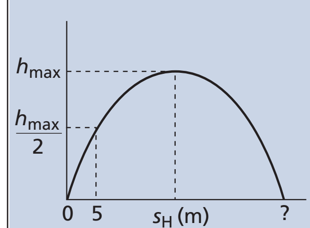
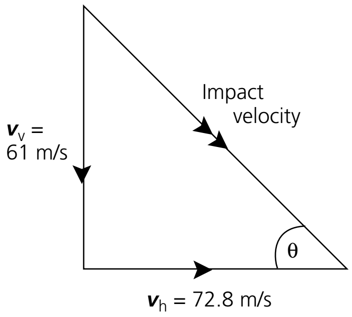


Figure 5.9(b)



The vertical component of the velocity will change:

$$\begin{aligned} v_v &= u_v + at \\ &= +50 + -10 \times 11.1 \\ &= -61 \text{ m s}^{-1} \text{ (downward)} \end{aligned}$$

The total velocity is the vector sum of the two components (Figure 5.9(**b**)). Using Pythagoras's theorem:

- Impact velocity = $\sqrt{61^2 + 72.8^2}$ = 95 m s^{-1}.
- Using trigonometric ratios: $\theta = \tan^{-1} \frac{61}{72.8} = 40°$.

(c) Horizontal distance (s_h) = horizontal component of velocity (v_h) × time of flight (t).

$$s_h = 72.8 \times 11.1 = 808 \text{ m from base of the cliff}$$

Note: you *can't* use the formula $R = \frac{u^2}{a} \sin 2\theta$ because the projectile is not landing at a position level with where it was thrown. The range formula is assuming the vertical displacement is zero.

Questions

3 A tennis ball close to the ground is hit by a racquet with a velocity of 30 m s^{-1} at an angle of 25° to the horizontal. Find **(a)** the initial vertical and horizontal components of the velocity; **(b)** the maximum height reached; **(c)** the time of flight; **(d)** the horizontal range.

4 A football is kicked off the ground at an angle of 30° to the horizontal. It moves away at 23.0 m s^{-1}. Calculate **(a)** the vertical velocity after 1.0 s; **(b)** the velocity of the ball after 1.0 s; **(c)** the maximum height reached; **(d)** the time of flight; **(e)** the range of the ball.

5 A rock is thrown off a 100.0 m cliff upward at an angle of 20° to the horizontal. If it has an initial velocity of 15 m s^{-1} and strikes the rocks below, calculate **(a)** the time of flight; **(b)** the impact velocity; **(c)** how far out from the base of the cliff the rock strikes the ground.

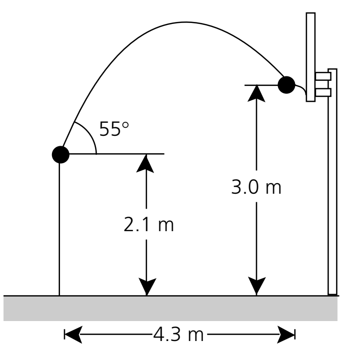
6 A difficult one! A basketball player shoots a ball at an angle of 55° into a hoop on a post 4.3 m away (Figure 5.10). If the ball is released from a height of 2.1 m and lands in the net, which is 10 feet (3.0 m) off the ground, calculate the initial speed of the ball for this foul shot to be successful.

7 Emmanuel Zacchini was a famous American 'human cannonball'. In 1940 he attempted to clear a Ferris wheel 18 m high after being launched from a cannon at an elevation angle of 53° and a muzzle velocity of 26.5 m s^{-1}.

(a) If his point of projection from the cannon was 3.0 m above the ground, did he clear the Ferris wheel?

(b) How far away from the cannon should the net have been placed?

Figure 5.10
For question 6.



The effect of air on projectiles

Aristotle argued that once a projectile ran out of impetus it would fall vertically from the sky. Galileo argued that this was wrong — the trajectory would be parabolic. Galileo was right — or was he? In the discussion so far we have ignored air resistance but when it is taken into account the trajectory is different. Aristotle is almost right but for the wrong reasons. At low speeds air resistance is negligible. But at greater speeds it becomes considerable. For instance, a flyball hit at an angle of elevation of 60° at 45 m s^{-1} will have different trajectories in air compared with those in a vacuum. Table 5.1 summarises the differences.

Table 5.1 TRAJECTORIES OF A BASEBALL

	PATH A (AIR)	PATH B (VACUUM)
Range	100 m	177 m
Maximum height	53 m	77 m
Time of flight	6.6 s	7.9 s

Figure 5.11 shows the difference between the trajectory of a ball as predicted by a computer model **(a)** and that of a bullet as tracked by ballistics experts on a rifle range **(b)**. The differences come about because bullets have more complicated motions than a round ball in flight.

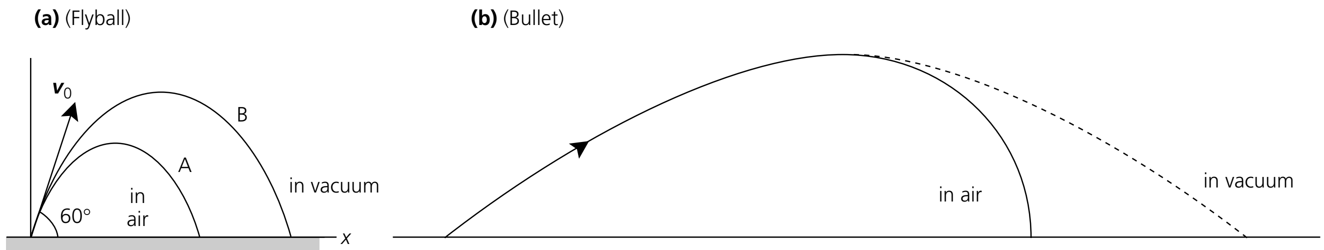


Figure 5.11
(a) The path of a flyball calculated taking air resistance into account (A) and in a vacuum (B).
(b) The dotted line is a trajectory of a bullet in a vacuum. The solid line shows how it is modified by air drag.

Trial-and-error has shown that the maximum range for a bullet fired in air is achieved at an elevation of 33°, a rough rule-of-thumb that works for most guns. As a crude approximation, the angle of descent is $2\frac{1}{2}$ times the angle of launch, so for a 33° elevation of fire, the bullet will arrive at 82.5°, or very nearly vertical. Any greater elevation of the gun merely means that the bullet will actually drop vertically and the last part of the flight will add nothing to the range. So the war veterans were probably right — bullets did fall on them vertically from the sky (and were just as lethal).

Exterior ballistics

Once a bullet leaves the muzzle of a gun, the laws of exterior **ballistics** take over as we have seen above. Ballistics comes from the Greek word *ballein* meaning 'to throw'. Modern high-speed photography enables physicists, chemists and engineers to study the explosion of propellant and the resulting motion of a projectile.

Typically, the bullet exits the muzzle at about 800 m s^{-1}, spinning at some 3000 revolutions per second. At first, it goes off down the range with a slight wobble, which straightens out after about 100 m, whereon it settles down to the main part of its flight, nose first, spinning steadily. This is the useful part of the bullet's life and it is intended that the bullet should hit its target during this stage. In the last part of its flight, the final slowing occurs and the bullet 'drops out of the sky'. The spinning tries to keep the bullet pointing straight ahead but as it falls toward Earth, the bullet cuts through the air sideways and air drag becomes great (Figure 5.12).

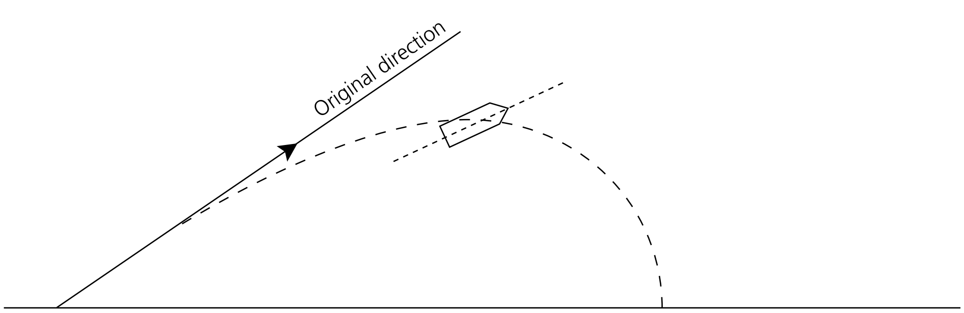


Figure 5.12
An exaggerated view of a wobbling (overstable) bullet, showing how it can fly almost broadside at the end of the trajectory.

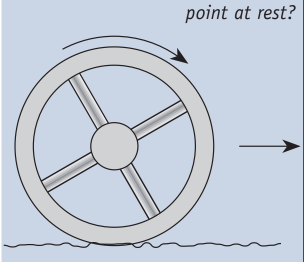
The bullet begins to tumble end-over-end and by this stage has a very unpredictable trajectory and is too unreliable. Rifles generally have an effective range of 400–900 m, although weapons like the AR15 Armalite (USA) are designed for modern jungle warfare and are only accurate to 450 m but have an enormous muzzle velocity of 990 m s^{-1} to compensate. Because the bullet is tumbling at the end of this distance, it tears apart whatever it hits.

UNIFORM CIRCULAR MOTION 5.4

It really wasn't until the 1500s that people began to believe that the Earth rotates on its own axis. Until then, the rate of rotation of objects was of little consequence. Today, rotation and its measurement is of fundamental importance to society, whether it is the rotation of microwave carousels, CDs, car tyres, engines, sewing machines, nuclei or orbiting satellites. In this section we will be looking at **circular motion**, that is, motion in a circle.

NOVEL CHALLENGE

When a wheel rolls along, is any point at rest?



A ball on a string

Imagine you are whirling a ball in a horizontal circle on a piece of string. By Newton's first law of motion, the ball is attempting to travel in a straight line but is stopped from doing so by your pull on the string. As your hand is at the centre of the circle in which the ball moves, the force on the string and hence on the ball is always towards your hand and hence towards the centre. This force is called a **centripetal** force (Latin *centrum* = 'centre', *petere* = 'seek'). When the object travels at constant speed in a circle, it is said to be undergoing **uniform circular motion**. Notice that its direction is continually changing so its velocity is not constant.

Figure 5.13

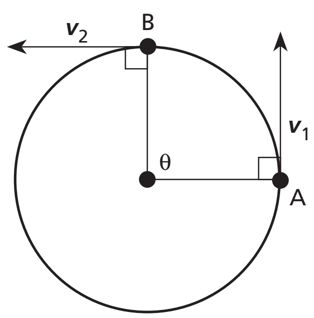
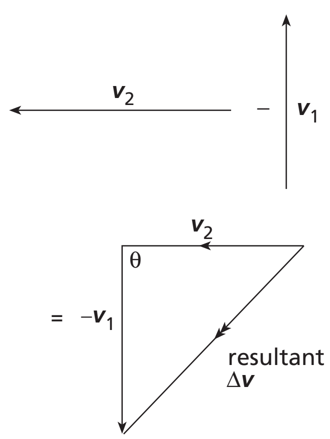


Figure 5.13 shows the motion of a ball moving in a circle of radius *r* at constant speed. The velocity at any point on the circle is a tangent to the path at that point. For instance, at position A, the velocity vector $\mathbf{v}_1$ points up the page. At point B, the velocity vector $\mathbf{v}_2$ points to the left but still has the same length as the speed remains the same. As the direction of the velocity has changed, the ball is said to be accelerating (**centripetal acceleration**). The magnitude and direction of this acceleration can be calculated by determining the change in velocity:

Change in velocity ($\Delta \mathbf{v}$) = final velocity ($\mathbf{v}_2$) − initial velocity ($\mathbf{v}_1$).

When we subtract a vector quantity, we turn it into an addition by reversing the direction of the initial vector and adding the arrows head to tail. Hence: $\Delta \mathbf{v} = \mathbf{v}_2 + {}^{-}\mathbf{v}_1$. As can be seen from Figure 5.14, the resultant is directed to the centre of the circle, hence 'centre seeking'.

Figure 5.14



Using similar triangles, it can be shown that the centripetal acceleration is given by:

$$a_c = \frac{v^2}{r}$$

where *r* is the radius of the circular path in metres. Note that the vector quantities *a* and *v* are no longer typed in **bold**. This is because they are not in the same direction. The acceleration is toward the centre whereas the velocity is at right angles to this.

The ball is experiencing a centripetal force to keep it moving in a circle. This is provided by the tension in the string. Using Newton's second law of motion ($\mathbf{F} = m\mathbf{a}$) we get:

$$\mathbf{F}_c = m\frac{v^2}{r}$$

A car going around a curve safely

A racing car travelling around a circular track is similar to a ball being whirled around on a string. A vehicle going round a bend on a level road can be viewed also as going on a circular path. The sideways friction between the tyres and the road provides the force needed to stop the car just going straight ahead. The friction provides the centripetal force. If the car hit a wet patch all of a sudden, the friction would be reduced and insufficient centripetal force could be provided so the car would tend to go straight ahead, possibly even spinning out of control.

Recall from earlier work that friction (F_f) is proportional to the force pressing the surfaces together (the normal reaction F_N): $F_f = \mu F_N$. On horizontal ground, the normal reaction is equal to the object's weight (F_w or mg).

If centripetal force is provided by the friction we can combine the two equations:

$$\boldsymbol{F}_c = \frac{mv^2}{r} \text{ and } \boldsymbol{F}_f = \mu \boldsymbol{F}_N = \mu m\boldsymbol{g}, \text{ then } \frac{mv^2}{r} = \mu m\boldsymbol{g}$$

$$\text{i.e. } v_{max} = \sqrt{\mu \boldsymbol{g} r}$$

The maximum safe speed to go around a curve is represented by $\boldsymbol{v}_{max}$ in the final equation above. The mass of the car doesn't come into the equation so in this case has no effect on the safe speed. Big cars have the same maximum safe speed as small cars.

Revolutions per second

You probably don't know the speed of the Moon about the Earth in metres per second or even kilometres per hour. But you would know that it makes one revolution in just over 27 days. Engine speeds too are usually expressed in a number of revolutions per minute (rpm). At idle, they might turn at 750 rpm and at cruising speed may reach say 4000 rpm. It depends on the car.

The distance covered in one revolution by an object in uniform circular motion at a distance r from the centre is equal to the circumference of the circle: $s = 2\pi r$. If the time taken to complete one revolution (called the **period**) is T, then:

$$v = \frac{s}{t} = \frac{2\pi r}{T}$$

This velocity is called the **tangential velocity** (Latin *tangere* = 'to touch'). It is sometimes called **radial** velocity. Angular velocities will be dealt with later.

Combining equations we get: $a_c = \dfrac{4\pi^2 r}{T^2}$ $\quad F_c = \dfrac{m4\pi^2 r}{T^2}$

Example

A motorcycle and rider with a total mass of 1250 kg are travelling around a circular track of radius 50 m at a constant speed of 40 m s^{-1}. Calculate **(a)** the centripetal acceleration; **(b)** the centripetal force; **(c)** the time it takes to complete one lap.

Solution

(a) $a_c = \dfrac{v^2}{r} = \dfrac{40^2}{50} = 32 \text{ m s}^{-2}$.

(b) $\boldsymbol{F}_c = m\dfrac{v^2}{r} = 1250 \times 32 = 40\,000$ N.

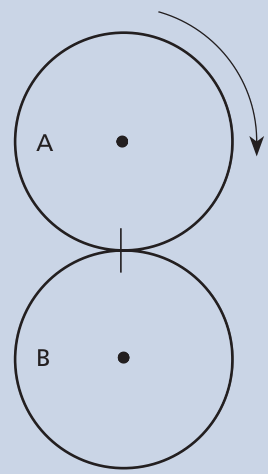
(c) $v = \dfrac{2\pi r}{T}$, or $T = \dfrac{2\pi r}{v} = \dfrac{2 \times 3.14 \times 50}{40} = 7.85$ s.

NOVEL CHALLENGE

Some coins were placed on a turntable in a line from the centre to the edge. The turntable was then turned on. *What do you predict will happen?*

NOVEL CHALLENGE

How many revolutions will coin A do while rotating around coin B? Try it. You'll be surprised.



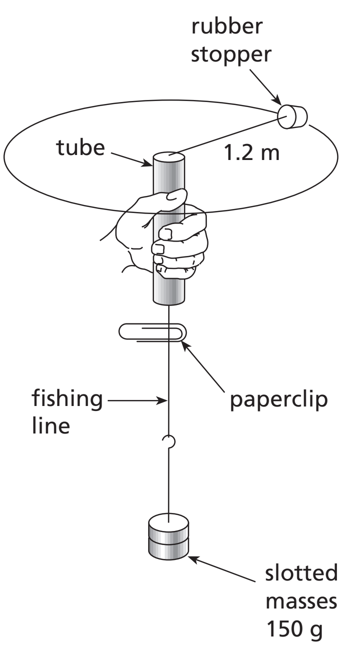
NOVEL CHALLENGE

The government steamer *Relief* attended the lighthouses along the Queensland coast from 1899 to 1952. To cope with the huge spray of seawater on the windows of the steering cabin, a novel approach was taken. The windscreen in part consisted of a circular glass disk about 40 cm diameter that spun at high speed.
How did this keep the seaspray off the window? Why couldn't they use windscreen wipers as in a car? Propose two advantages and two disadvantages of this sytem compared with wipers.

EI Activity 5.4 THE WHIRLING STOPPER

Tie a rubber stopper to a piece of string and whirl it in a horizontal circle above your head. See if you can let it go so that it will hit the wall of your room at right angles. Whereabouts in its circular travel did you have to let it go to achieve this? Which law of motion is confirmed by this?

Figure 5.15



Example 1

In an investigation of uniform circular motion, a student whirled a 50 g rubber stopper above his head in a horizontal circle of radius 1.2 m (Figure 5.15).

The string was passed through a piece of glass tubing and a set of slotted brass masses was suspended off the end of the string. It required 150 g of hanging mass to provide enough force to keep the rubber stopper whirling in a circle at a constant speed. Use $\boldsymbol{g}$ = 9.8 m s^{-2} and calculate **(a)** the centripetal force provided by the hanging mass; **(b)** the tangential velocity of the stopper; **(c)** the period of the rubber stopper; **(d)** the time taken for 10 revolutions of the stopper.

Solution

(a) The centripetal force is provided by the weight of the hanging mass:

$$\boldsymbol{F}_c = \boldsymbol{F}_w = m\boldsymbol{g} = 0.150 \text{ kg} \times 9.8 \text{ m s}^{-2} = 1.47 \text{ N}$$

(b)

$$\boldsymbol{F}_c = \frac{m\boldsymbol{v}^2}{r}, \text{ or } \boldsymbol{v}^2 = \frac{\boldsymbol{F}_c r}{m} = \frac{1.47 \text{ N} \times 1.2 \text{ m}}{0.050 \text{ kg}} = 35.3$$

$$\boldsymbol{v} = \sqrt{35.3} = 5.9 \text{ m s}^{-1}$$

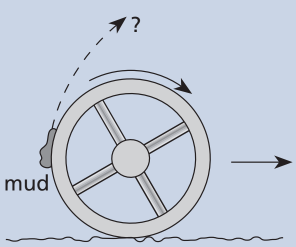
(c)

$$T = \frac{2\pi r}{\boldsymbol{v}} = \frac{2 \times 3.14 \times 1.2}{5.9} = 1.3 \text{ s}$$

(d) Time for 10 revolutions = 10 revolutions × 1.3 s/rev = 13 s.

NOVEL CHALLENGE

A wheel is rolling along with constant speed and a lump of mud is thrown off its hindmost point.
Will it touch the wheel again?



Example 2

A car of mass 1750 kg is rounding a curve of radius 70 m at a speed of 20 m s^{-1}. The surface is dry and the coefficient of friction between the tyres and the road is 0.65. The driver then hits a wet patch on the curve where the coefficient of friction is 0.25. Calculate **(a)** how much below the safe maximum speed the car is doing on the dry section of the curve; **(b)** whether the driver has to slow down to safely travel along the wet section and, if so, to what safe maximum speed; **(c)** would a smaller and lighter car allow the driver to go faster around the curve?

Solution

(a) $\boldsymbol{v}_{max} = \sqrt{\mu \boldsymbol{g} r} = 0.65 \times 10 \times 70 = 21.3$ m s^{-1}; the driver is 1.3 m s^{-1} below this speed.
(b) $\boldsymbol{v}_{max} = \sqrt{\mu \boldsymbol{g} r} = 0.25 \times 10 \times 70 = 13.2$ m s^{-1}; the driver has to slow down to this speed.
(c) $\boldsymbol{v}_{max}$ is independent of mass, so a lighter car would make no difference.

Cambered surfaces

Some curved motor car racing tracks are **cambered** or **banked**, that is, tilted in towards the centre of the curve. In this case the component of the vehicle's weight down the slope helps to provide centripetal force so the frictional force need not be as great. Alternatively, the car can safely travel at much higher speeds. Road engineers often camber roads the 'wrong' way for purposes of drainage. You could imagine the effect this has on maximum safe speeds?

Some other examples

- For a **space shuttle** and satellites orbiting the Earth or planets orbiting the Sun, the centripetal force is provided by gravitational forces. This will be dealt with in a later chapter.

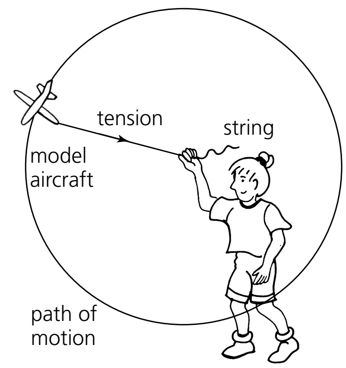
- In a **gravitron** or rotor at an amusement park, the person is 'pressed' against the wall. Actually, the person is trying to travel in a straight line but the wall pushes on the person (the centripetal force) and the person pushes back. The centripetal force is the normal force directed radially inward on the rider. At high speeds, this normal force becomes sufficiently great to provide enough friction to stop the rider sliding down the wall under the influence of gravity.
- A **spin-dryer** works on a centripetal force principle. When the tub is spun at high speed, the force of attraction between the water and the clothes is insufficient to keep the water moving in a circle. The liquid moves tangentially and out through the holes in the sides of the tub.

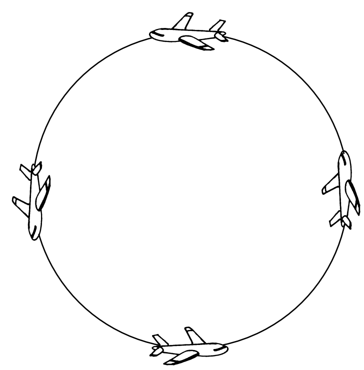
SR Activity 5.5 SPIN-DRYER CHAMPIONSHIPS

1 Spin-dryers go pretty fast — too fast to see with the naked eye. Design a method to measure the speed of a spin-dryer in revolutions per minute (rpm). You don't have to build it or have the parts at home or school; just design the procedure and instrumentation.

2 If your method is simple, do it and report the result to the class.

Figure 5.16
Looping the loop in a vertical circle.





Questions

8 A car of mass 1900 kg is travelling at a constant speed of 25 m s^{-1} around a level corner of radius 50 m. Calculate **(a)** the centripetal acceleration; **(b)** the centripetal force acting on the car.

9 An aeroplane is travelling at 200 m s^{-1} in a circular path of radius 3000 m. Calculate **(a)** the centripetal acceleration of the plane; **(b)** the time to complete one revolution.

10 The Moon takes a period of 27.3 days to complete one orbit of the Earth. If we consider the path to be circular then its average radius is 3.84×10^8 m from the centre of the Earth. Determine **(a)** the circumference of the Moon's path; **(b)** the speed of the Moon; **(c)** the Moon's centripetal acceleration; **(d)** the centripetal force (the Moon's mass is 7.34×10^{22} kg).

11 Spin-dryers revisited:
(a) Why do clothes that comes out of a spin-dryer still feel damp?
(b) Would continued spinning at the same speed get rid of more water?
(c) Could you spin them completely dry?
(d) How does the water get from the clothes in the middle to the outside (is there a more efficient way)?

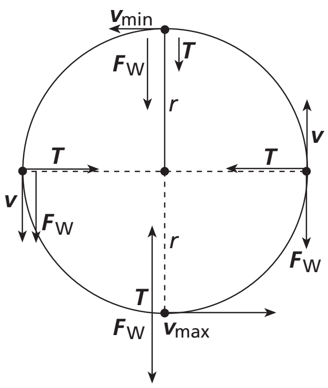
12 A mass of 150 g is whirled in a horizontal circle of radius 95.0 cm on a string. If 10 revolutions at constant speed take 4.5 seconds, calculate the tension in the string.

5.5 NON-UNIFORM CIRCULAR MOTION

The previous section dealt with uniform circular motion. This can be easily achieved by objects travelling in horizontal circles. When they travel in vertical circles it is difficult to keep the speed constant and this is called **non-uniform** circular motion. Two common examples of this are a ball on a string and an aircraft loop-the-loop. Devices that have stiff radial arms such as a bicycle wheel, a ferris wheel or a pulley cannot be considered non-uniform as they are completely rigid and all points on the circumference travel at the same speed.

When a ball is swung on the end of a string in a vertical circle, the speed of the ball is greatest at the bottom of the circle and slowest at the top of the circle. Hence, the centripetal acceleration is smallest at the top and greatest at the bottom. In the following, the symbol $\boldsymbol{T}$ is used to represent the tension in the string, whereas $\boldsymbol{F}_w$ represents the weight of the ball (= mg). Refer to Figure 5.17.

Figure 5.17



- At the top, the string doesn't have to pull as hard ($\boldsymbol{F}_c$) because the weight is helping it pull down:

$$\boldsymbol{F}_c = \frac{m\boldsymbol{v}^2}{r} = \boldsymbol{T} + \boldsymbol{F}_w \text{ or } \boldsymbol{T} = \boldsymbol{F}_c - \boldsymbol{F}_w$$

- At the side, the weight has no effect on the tension:

$$\boldsymbol{F}_c = \frac{m\boldsymbol{v}^2}{r} = \boldsymbol{T}$$

- At the bottom, the string has to pull harder because it has to support the weight of the ball as well:

$$\boldsymbol{F}_c = \frac{m\boldsymbol{v}^2}{r} = \boldsymbol{T} - \boldsymbol{F}_w \text{ or } \boldsymbol{T} = \boldsymbol{F}_c + \boldsymbol{F}_w$$

The apparent weight of the ball at the top or bottom is given by $\boldsymbol{T}$.

Minimum velocity

The minimum velocity needed to keep a ball in a circular orbit is found to be the velocity at the instant when the string begins to slacken (i.e. when $\boldsymbol{T} = 0$) at the top. This is when:

$$\frac{m\boldsymbol{v}_{min}^2}{r} = \boldsymbol{T} + \boldsymbol{F}_w = 0 + \boldsymbol{F}_w = m\boldsymbol{g}$$

$$\boldsymbol{v}_{min}^2 = \boldsymbol{g}r \text{ so } \boldsymbol{v}_{min} = \sqrt{\boldsymbol{g}r}$$

Maximum velocity

The maximum velocity occurs at the bottom of the path:

$$\frac{m\boldsymbol{v}_{max}^2}{r} = \boldsymbol{T} - \boldsymbol{F}_w$$

INVESTIGATING

Many factories, laboratories and industries use centrifuges. Locate two places that use centrifuges and write a report comparing and contrasting their uses and performances.

Example 1

The breaking strain of a string is 50 N. A 250 g ball is whirled in a vertical circle of radius 1.2 m. Calculate **(a)** the minimum velocity needed to keep the ball in orbit; **(b)** the maximum speed that the ball can have before the string breaks.

Solution

(a) $\boldsymbol{v}_{min} = \sqrt{\boldsymbol{g}r} = \sqrt{10 \times 1.2} = 3.5$ m s^{-1}.

(b) Maximum tension occurs at the bottom of the ball's path:

$$\frac{m\boldsymbol{v}_{max}^2}{r} = \boldsymbol{T} - \boldsymbol{F}_w = 50 - 0.25 \times 10 = 47.5 \text{ N}$$

$$\boldsymbol{v}_{max}^2 = \frac{47.5 \times 1.2}{0.25} = 228, \text{ hence } \boldsymbol{v}_{max} = \sqrt{228} = 15 \text{ m s}^{-1}$$

Example 2

A stunt pilot is diving his plane vertically downwards at a velocity of 200 m s^{-1} when he pulls out of the dive and changes his direction to a circular path of radius of 1000 m. If his mass is 70 kg and he continues to maintain constant speed in the circle, **(a)** what is the maximum centripetal acceleration he experiences; **(b)** what is the maximum force that his seat will exert on him? **(c)** If pilot 'blacks-out' when the acceleration is greater than 3$\boldsymbol{g}$, will he stay conscious? **(d)** At what circular path radius would he be liable to black out?

Solution

(a) Maximum acceleration (at bottom of path) $\boldsymbol{a}_c = \frac{\boldsymbol{v}^2}{r} = \frac{200^2}{1000} = 40\ \text{m s}^{-2}$.

(b) At bottom of loop, the seat provides the equivalent of the tension:

$$\frac{m\boldsymbol{v}^2_{max}}{r} = \boldsymbol{T} - \boldsymbol{F}_w,\ \text{hence}\ \boldsymbol{T} = \frac{m\boldsymbol{v}^2_{max}}{r} + \boldsymbol{F}_w = \frac{70 \times 200^2}{1000} + 70 \times 10 = 3500\ \text{N}$$

(c) $\boldsymbol{a}_c = 40\ \text{m s}^{-2}$. The number of '*g*' this is equal to is $40\ \text{m s}^{-2} \div 10\ \text{m s}^{-2} = 4$ '***g***'. This is greater than 3***g*** so the pilot will black out.

(d) To achieve 3***g*** (30 m s^{-2}), the radius can be calculated:

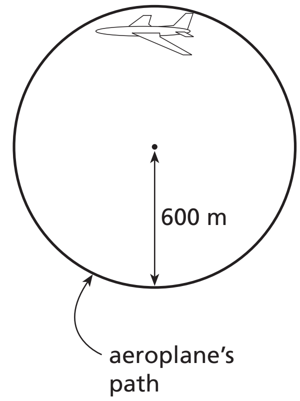
$$\boldsymbol{a}_c = \frac{\boldsymbol{v}^2}{r},\ \text{hence}\ r = \frac{\boldsymbol{v}^2}{\boldsymbol{a}_c} = \frac{200^2}{3 \times 10} = 1330\ \text{m}.$$

Even at this radius, the pilot may black out for a few seconds. Too tight a loop or too high a speed could cause the pilot (and crew) to black out for much longer. This could cause lack of control of the aircraft, death, or both. Some stunt!

Modern military aircraft typically have ***g*** limits of around +9.5 to −5.5 ***g***, although these boundaries are continually being pushed back. Sensors are fitted into most cockpits to allow the pilot to monitor ***g*** values to avoid overstressing the airframe. For additional safety and to cope with crash impacts, cockpit interiors are designed to withstand 20 ***g*** in any direction. Ejector seats and escape pods may suffer instantaneous loads (for about 0.1 s) in excess of 30 ***g***. The requirement is that a seat shoots a pilot from an aircraft at zero forward speed and zero altitude (the so-called 'zero–zero' seat) to an altitude at which the parachute can open safely. Alternatively, the seat must be able to clear the tailplane of an aircraft travelling at high speed. The record for a human experiencing ***g***-loading is around 86 ***g*** by the occupant of a rocket-sled. By comparison, civilian airlines experience a modest 1.5 ***g*** during take-off acceleration.

The world record for loops is held by David Childs (USA). He did 2368 loops in a Bellanca Decathlon plane over the North Pole on 9th August 1986. Imagine having 'Crazy Dave' in your physics class.

Figure 5.18 For question 13.



Questions

13 A pilot is performing aerial acrobatics at an air show. He drives around a vertical loop of radius 600 m (Figure 5.18). What is the minimum speed at the top of the loop?

14 A 75 kg pilot flies his plane in a vertical circle of radius 600 m and at the bottom of the loop he is travelling at 120 m s^{-1}.

(a) What is the force of the seat on the pilot at this point?

(b) What is the acceleration in m s^{-2}?

(c) If he is known to black out at 5 '*g*', would he black out at the bottom of the loop?

(d) If the plane was travelling at 80 m s^{-1} at the top of the loop, what would the force of the seat on the pilot be?

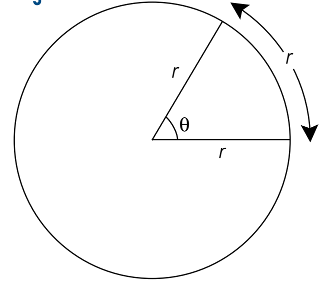
(e) At what speed would the plane have to travel for the pilot to be just weightless at the top of the loop, that is, his weight equals the centripetal force?

5.6 ANGULAR VELOCITY

When something completes one revolution it has gone through 360°. One revolution per second is the same as 360° per second. This is called its **angular velocity**.

In maths, you may have measured angles in **radians**. One radian (rad) is the angle when the arc length is the same as the radius of the circle (Figure 5.19). There are 2π radians in a circle of 360°, thus one revolution equals 2π radians. Angular velocity (ω) is usually

Figure 5.19



θ = 1 rad
1 rev = 2π rad

expressed in radians per second (rad s^{-1}). It is a vector quantity. The symbol ω is the Greek letter 'omega'. The word radian comes from the Latin *radius*, meaning the spoke of a wheel.

Tangential velocity (***v***) = angular velocity (ω) × radius (r).

$$\boldsymbol{v} = \omega r \text{ or } \omega = \frac{\boldsymbol{v}}{r}$$

Centripetal acceleration and force can also be expressed in terms of angular velocity:

$$\boldsymbol{a}_c = \frac{\boldsymbol{v}^2}{r} = \omega^2 r = \omega \boldsymbol{v} \text{ and } \boldsymbol{F}_c = \frac{m\boldsymbol{v}^2}{r} = m\omega^2 r$$

The period T of a rotating object is given by: $\omega = \frac{2\pi}{T}$ or $T = \frac{2\pi}{\omega}$.

Why angular velocities?

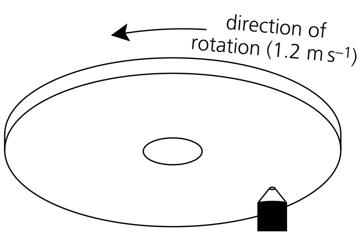


Figure 5.20
An underside view of a CD showing the objective lens of the laser pickup.

You may wonder what the point of using angular velocities is. A spinning disc such as a CD (Figure 5.20) will have all points on the surface turning at the same angular speed even though different tracks will have different linear velocities. It makes the speed easier to state. Another common way of expressing angular speeds is **revolutions per minute (rpm)**. A microwave carousel does about 5 rpm.

Example 1

A tyre is turning at 20 m s^{-1} as a car travels along a road. If the diameter of the tyre is 62 cm, calculate **(a)** the angular velocity of the tyre; **(b)** the centripetal acceleration of a 2 g stone embedded in the tread of the tyre; **(c)** the centripetal force acting on the stone; **(d)** the rate of rotation of the tyre in rpm.

Solution

(a) Radius = 0.31 m; $\omega = \frac{\boldsymbol{v}}{r} = \frac{20}{0.31} = 64.5$ rad s^{-1}.

(b) $\boldsymbol{a}_c = \frac{\boldsymbol{v}^2}{r} = \frac{20^2}{0.31} = 1290$ m s^{-2}.

(c) $\boldsymbol{F}_c = m\boldsymbol{a}_c = 0.002 \times 1290 = 2.58$ N.

(d) 1 revolution = 2π radians; hence number of revolutions per second = $\frac{64.5 \text{ rad s}^{-1}}{2\pi}$ = 10.2 rps = 616 rpm.

Example 2

A flywheel of radius 2.0 m is rotating at 120 rpm. Calculate **(a)** the angular velocity; **(b)** the linear velocity of a point on the rim.

Solution

(a) 1 rpm = 2π rad min^{-1}; hence 120 rpm = $120 \times 2\pi$ rad min^{-1} = $\frac{120 \times 2\pi}{60}$ rad s^{-1} = 4π rad s^{-1}.

(b) $\boldsymbol{v} = \omega r = 4\pi \times 2 = 25$ m s^{-1}.

Photo 5.1
A tachometer. Note the 'red line' from 5 to 7 thousand revs per minute.

Everyday examples of angular motion

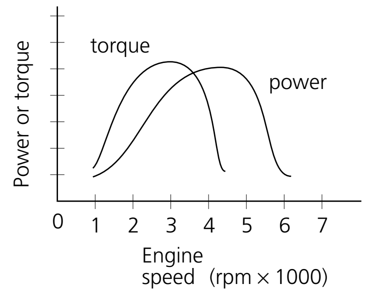
- **Record players** generally have three speeds: 78 rpm for the old bakelite 78s; 45 rpm for vinyl singles and $33\frac{1}{3}$ rpm for LPs. As angular speeds were constant for any particular record, the outside track of a 12 inch (30 cm) LP travelled at a higher linear speed than the inside track, so the outside track gave better sound reproduction. For instance, the outside track at a radius of 14.5 cm had a linear speed of 50 cm s^{-1}, whereas the inside track at a radius of 6.5 cm gave a linear speed of 22 cm s^{-1}.

To overcome the problem of differential track speeds, when compact disc players were developed the track speed was kept constant and the rotation speed was varied. For example, the linear speed of a CD is about 1.2 m s^{-1}, so for an outside track (radius 58 mm), the rotation rate is 200 rpm, whereas for an inside track (r = 23 mm), the rotation rate is 500 rpm (see Figure 5.20). Computer disk drives work on the same principle.

- **Car engines** generally idle at about 800 rpm and cruise at somewhere between 2000 and 4000 rpm. Cars with V8 engines generally have more power and torque (turning force) than either six or four cylinder cars so they can cruise at lower engine speeds. It is unusual for car engines to rev above 6000 rpm because the valves and other components can be damaged. Sportier cars are sometimes equipped with tachometers (Latin, *tachy* = 'swift'), which measure engine speeds in rpm. The maximum recommended speed is indicated with a red line and if you '**red-line**' an engine you are certainly giving it a good thrashing. The power and torque delivered by engines is not constant over the full range of engine speeds (Figure 5.21). Cars are geared so that drivers can maintain the engine speed just below the optimum power and torque range, which usually corresponds to the normal cruising speed in top gear. For instance, a Toyota Landcruiser with a 4.5 L, six cylinder petrol engine develops maximum power at 4600 rpm and maximum torque at 3200 rpm. At a cruising speed of 100 km h^{-1}, the engine turns over at a relatively slow 2100 rpm, leaving plenty of revs in reserve for overtaking.

  The same is true of motorcycles except that they run at much higher revs; a range of 6000 rpm to a red line at 12 000 rpm is typical.

Figure 5.21



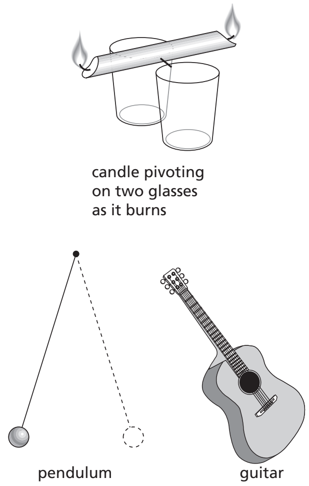
Questions

15 Calculate the force acting on a mass of 3 kg that is rotating at 5 rad s^{-1} in a circle of radius 30 cm.

16 A microwave oven carousel has a diameter of 40 cm and does one revolution in 12 seconds. Calculate **(a)** the angular velocity of the carousel; **(b)** the tangential velocity.

17 While reading the fifth song on a CD, the laser pickup diode is at a radial distance of 50 mm from the centre of the spinning disc. If the linear velocity of the disc directly above the laser pickup is 1.2 m s^{-1}, calculate the angular velocity in **(a)** rad s^{-1}; **(b)** rpm.

Figure 5.22
Three examples of periodic motion.



5.7 SIMPLE HARMONIC MOTION

A swinging pendulum, a vibrating guitar string and a mass **oscillating** on the end of a spring are all examples of **periodic motion** — motion in which an object continually moves back and forth over the same path in equal time intervals (Figure 5.22). The word *oscillate* means to move back and forth. It comes from the Latin *os*, meaning 'mouth' or 'face'. The Greeks used masks of the god Bacchus hung up as charms in vineyards and they swung back and forth in the breeze, hence os-cillate.

Not all periodic motions are as simple as a mask blowing in the bleeze; some are very complex. However, in this chapter we will look at a simple type of periodic motion called **simple harmonic motion** (SHM).

The vibrating mass

Figure 5.23

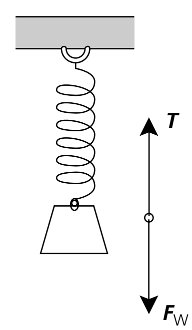


Figure 5.23 shows a mass attached to a spring hooked to the ceiling. When it is at rest, the tension in the spring and the weight are equal and opposite — or equally balanced. This position is called the **equilibrium** position (*equi* = 'equal', *libra* = 'balance'). There is no net

force so the mass is not accelerating. The displacement of the mass from the equilibrium position is also called the **amplitude** (*x*) and is zero in this position.

If the mass is pulled down and let go it oscillates up and down as shown in Figure 5.24. A study of the forces and displacements is quite revealing.

Figure 5.24

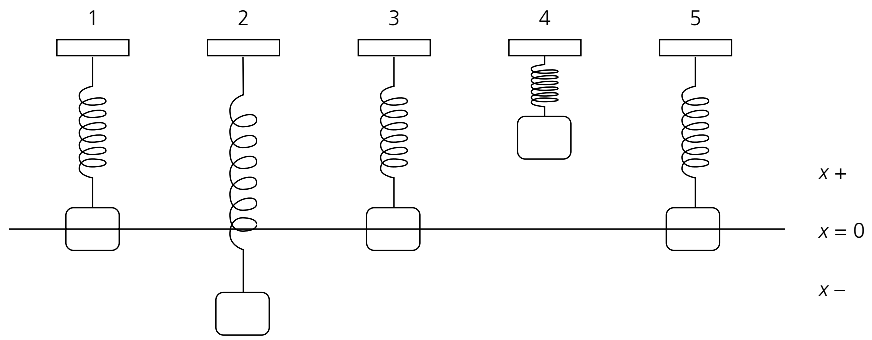


Table 5.2 summarises the variables involved.

Table 5.2

	POSITION 1	POSITION 2	POSITION 3	POSITION 4	POSITION 5
Net force	0	max. up	0	max. down	0
Acceleration	0	max. up	0	max. down	0
Velocity	max. down	0	max. up	0	max. down
Displacement	0	max. down	0	max. up	0

- Position 1 — the mass is moving downward through its equilibrium position so the net force is zero but it is moving with maximum speed. As there is no net force, the acceleration must also be zero (Newton's second law: $\boldsymbol{F} \propto \boldsymbol{a}$).
- Position 2 — the mass is at its lowest point so displacement is a maximum in the downward or negative direction. The spring is stretched so the tension in it is greater than the weight of the object so the net force is directed upward (positive). Acceleration is also directed up.
- Position 3 — the mass is back to its equilibrium position but is now moving with maximum velocity upward.
- Position 4 — the spring is now unstretched so the tension in the spring is zero. The only force comes from the weight so the net force is a maximum in the downward (negative) direction. Displacement is a maximum in the positive direction.
- Position 5 — equilibrium, with the object moving down at maximum speed.

In summary:

- Simple harmonic motion (SHM) is periodic motion in which $\boldsymbol{F} \propto -\boldsymbol{x}$.
- When the force ($\boldsymbol{F}$) is a maximum, the displacement ($\boldsymbol{x}$) is a maximum but in the opposite direction.
- When the force is a minimum (zero), the displacement ($\boldsymbol{x}$) is also a minimum (zero).

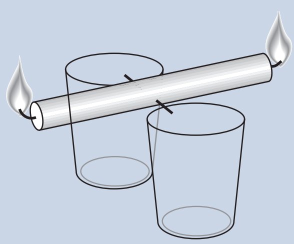
Mathematically:

$$\boldsymbol{F} \propto -\boldsymbol{x} \text{ or } \boldsymbol{F} = -k\boldsymbol{x}$$

The constant (k) is called the **spring constant**. Its units will be N m^{-1}. The stiffer the spring the larger the spring constant.

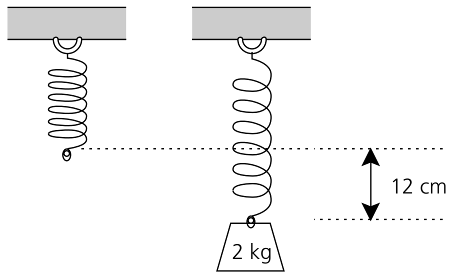
NOVEL CHALLENGE

A candle with a nail through the middle is supported on two glasses and lit at both sides. *How could you check if the resulting motion is SHM or just periodic?*



Example

Figure 5.25



When a mass of 2.0 kg is attached to a spring it stretches by 12 cm (Figure 5.25).

(a) Calculate the spring constant.

(b) What would the stretch be if a further 1.0 kg was added?

Solution

(a) $\boldsymbol{F} = -k\boldsymbol{x}$ or $k = -\frac{\boldsymbol{F}}{\boldsymbol{x}} = -\frac{20}{0.12} = 167$ N m^{-1}.

(b) $\boldsymbol{x} = \frac{\boldsymbol{F}}{-\boldsymbol{k}} = \frac{30}{-167} = 0.18$ m = 18 cm.

Experiments show that if the spring is of negligible mass compared with the object hanging on it, then the period (T) of the motion is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Example

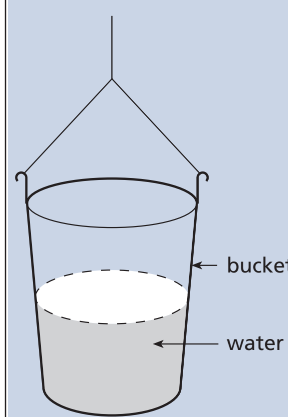
A light spring has a mass of 100.0 g attached to it. If it has a spring constant of 4.5 N m^{-1}, calculate the period of the vibrating spring.

Solution

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.1000}{4.5}} = 0.93 \text{ s}$$

NOVEL CHALLENGE

A pendulum has a bucket for the bob and it is half-filled with water. *When it freezes, predict what happens to the period of the pendulum.*



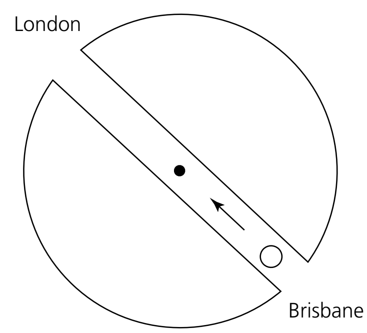
Journey to the centre of the Earth

An idea that has intrigued people for years is a hole through the Earth. Imagine a hole from Brisbane to London — it would be about 1.3×10^7 m long (Figure 5.26). If you dropped a parcel in one end it would come out the other some time later. A 1 kg parcel dropped into the hole at Brisbane would experience an initial force due to gravity of 10 N and would be pulled to the centre of the Earth some 6.5×10^6 m away. SHM would apply and we could calculate the force constant (k) = $\frac{\boldsymbol{F}_w}{\boldsymbol{x}} = \frac{10}{6.5 \times 10^6} = 1.5 \times 10^{-6}$ N m^{-1}.

Using the SHM formula: $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1}{1.5 \times 10^{-6}}} = 5066$ s (for one oscillation).

The time to get to the other side of the Earth would be half that or 2532 s (= 42 minutes).

Figure 5.26



Questions

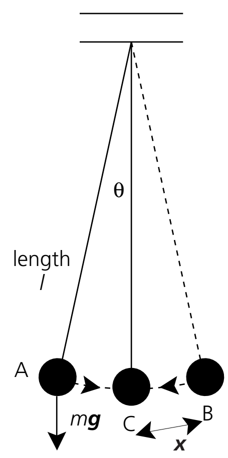
18 What assumptions have been made in the above example about the hole through the Earth that would make it an impossibility to achieve? List as many as you can.

19 A light spring has a mass of 200 g attached to it. When it is set oscillating, its period is measured to be 1.2 s. Calculate its spring constant.

20 To measure the mass (M) of an astronaut in the weightless conditions of space, an oscillating chair (mass m) bound to a spring is used. The body mass measuring device (BMMD) has a period of oscillation of 0.901 49 s when no one is in it. When one of the Skylab astronauts sat in it its period increased to 2.088 32 s. If the spring constant for the BMMD is 605.6 N m^{-1}, calculate the mass of the chair and of the astronaut.

The pendulum

Figure 5.27

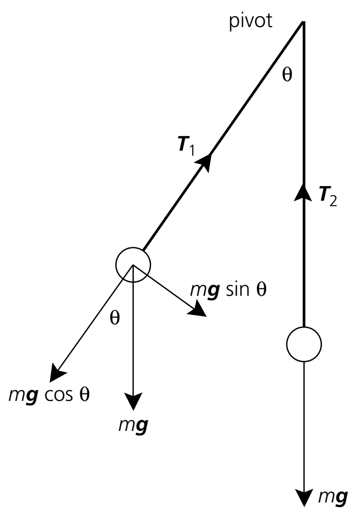


If you hang an apple on the end of a long thread fixed at its upper end, and then set it swinging, you can see that the motion is periodic (see Figure 5.27). It is also simple harmonic motion. Such an arrangement is called a **pendulum** (Latin *pendulus* = 'swinging'). The weight on the end is called the 'bob'. Why 'bob'? It comes from the Old French *bober*, meaning 'to mock'. When you mock someone your head moves up and down as you laugh.

As the pendulum, of length l, moves from A to B and back again to A, it makes a complete oscillation. The time required is the **period** (T). The number of oscillations per second is called its **frequency** (f). The sideways displacement ($\boldsymbol{x}$) is the sideways distance from the vertical or equilibrium position. The maximum displacement during the oscillations is called the **amplitude** (*amplus* = 'large'). The position C is called the **equilibrium** position. The forces acting on a pendulum during its travel are shown in Figure 5.28.

At an angle of θ as shown, the restoring force is equal to the component of the weight (= $m\boldsymbol{g}$) directed back to the equilibrium position (= $m\boldsymbol{g}\sin\theta$). The tension in the string ($\boldsymbol{T}_1$) is equal to the component (= $m\boldsymbol{g}\cos\theta$). At the equilibrium position, the restoring force is zero as θ equals zero and the component of the weight pulling the bob sideways is therefore also zero ($\sin 0° = 0$). The tension in the string ($\boldsymbol{T}_2$) is now equal to $m\boldsymbol{g}$ as $\cos 0° = 1$. The tension $\boldsymbol{T}_2$ is greater than $\boldsymbol{T}_1$.

Figure 5.28



The pendulum formula

Experiments show that the period of a pendulum is given by:

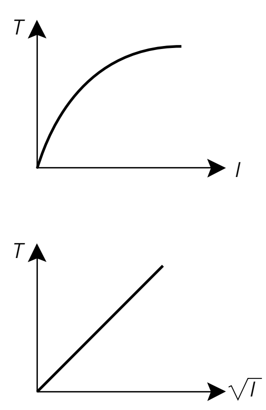
$$T = 2\pi\sqrt{\frac{l}{g}}$$

Note that the period is independent of the mass of the bob and amplitude (if it is fairly small, e.g. less than 20°) but as the graphs in Figure 5.29 show, T is proportional to $\sqrt{l}$.

Galileo is said to have confirmed that the period of a pendulum is independent of its amplitude. He observed the gentle swaying of a sanctuary lamp in the cathedral at Pisa. Using his pulse as a timer he found that successive oscillations were made in equal times, regardless of the amplitude. He later verified these observations in his laboratory.

The simple pendulum can be used to calculate $\boldsymbol{g}$ at any place by measuring T and l for a pendulum oscillating at that place. Countless thousands of such measurements have been made in the course of geophysical prospecting.

Figure 5.29



Example

The period of a simple pendulum 50.0 cm long is 1.42 s. Determine the acceleration due to gravity at that location.

Solution

$$T = 2\pi\sqrt{\frac{l}{g}}, \text{ or } T^2 = 4\pi^2\frac{l}{g}$$

$$\text{Hence } \boldsymbol{g} = \frac{4\pi^2 l}{T^2} = \frac{4\pi^2 \times 0.50}{1.42^2} = 9.79 \text{ m s}^{-2}$$

EI Activity 5.6 A SPRING PENDULUM

Make a pendulum out of a spring instead of a piece of string. Set it swinging and you'll soon see that T_2 is greater than T_1 as it bobs up and down as well as oscillating back and forth. The motion is fascinating. It almost makes you go to sleep.

EI Activity 5.7 THE SWEET SPOT

Any object that can vibrate like a pendulum is called a **physical pendulum** as distinct from a **simple pendulum**, which is a bob on a string. A wooden ruler, a cricket bat and a squash racquet can oscillate back and forth if allowed to pivot.

1 Suspend a **metre ruler** on a pin or nail through the hole in its end. Make sure it can vibrate freely. Set it in motion and measure the time it takes to make 10 swings. Calculate its period (T) and then calculate the effective length (l) using the pendulum formula. Assume g = 9.8 m s^{-2}. Mark this distance on the ruler. It is probably at about the 60 cm mark. This is called the centre of oscillation or centre of percussion (Latin *percussio* = 'striking'). You'll see why in the next part.

2 Repeat the above but use a **cricket bat** this time. Use two pins stuck into the handle at a point where your main grip would be and suspend the bat between the backs of two chairs. The pins can act as a pivot (Figure 5.30). Time it for 10 swings and calculate the effective length. Mark the centre of percussion (P). This is also called the 'sweet spot' because there is no sting in your hands if you hit the ball at this point. If the ball hits at any other point, the bat rotates about some other point than P, which accounts for the sting.

3 Try the same for a **squash racquet**. An effective length of 49 cm is common, which puts the sweet spot right at the middle of the head area. However, why do some world champion players hold their racquets where the grip joins the shaft? The answer is that the racquet is not rigid but flexes like a guitar string about the midpoint of the shaft while the end of the handle stays still. That's where they grip it to avoid the jarring. But designers also have to consider the power centre — the point at which maximum power is transferred to the ball. This is another complication that also applies to cricket, baseball and softball bats. This will be discussed further in Chapter 8, Momentum.

4 Over the past few years the sweet spot in **tennis racquets** has become higher up the head of the racquet. As a result, players can reach higher for the ball when they serve, opening up more of the opponent's court. This is a huge advantage because players can smack the ball that much harder instead of aiming carefully. For example, the world's fastest servers can now reach more than 200 km/h — speeds that were unheard of several years ago. Commentators have argued that speeds over 200 km/h are basically unplayable (and therefore boring) and that tennis balls need a 20% diameter increase to slow the maximum speed to a playable 180 km/h. If you can get hold of an old tennis racquet and a new one, compare the position of the sweet spots by the pendulum method. Is the above assertion correct?

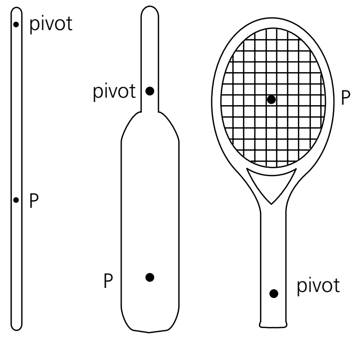


Figure 5.30
The 'sweet spot'. The centre of percussion can be measured experimentally.

NOVEL CHALLENGE

Two side-by-side pendulums are oscillating. One has a period of 6 s and the other a period of 7 s.

If the bobs are touching at one time, how much longer must you wait until they come together again?

NOVEL CHALLENGE

You have been asked by your employer to write an instruction manual for a swing set in which you have to explain how a user can make it go higher.

What would you say? Now explain the physics behind your instructions.

Questions

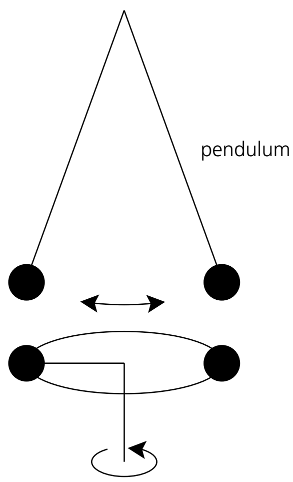
21 Determine the period of a pendulum with a length of 67.2 cm at a place where **(a)** g = 9.81 m s^{-2}; **(b)** g = 9.78 m s^{-2}.

22 **(a)** If you were accelerating upwards in a lift at 1.5 m s^{-2} what would the apparent acceleration due to gravity be?
(b) What would the period of oscillation of a 30 cm pendulum be in this lift?
(c) If the period of oscillation was 0.95 s, what acceleration upward would the lift be undergoing?

SHM AND CIRCULAR MOTION COMPARED 5.8

There is a very clear relationship between SHM and circular motion. Galileo was the first person to make observations in this respect. In 1610 he was using his newly constructed telescope and discovered the four principal moons of Jupiter. Over weeks of observation, each moon seemed to be moving back and forth past the planet in what we now call simple harmonic motion. This has been confirmed by plotting his data. But actually, the moons move in an essentially constant circular motion around Jupiter. What Galileo saw — and what you can see with a pair of binoculars — is this circular motion edge on, and they look as though they are oscillating back-and-forth beside the planet.

Figure 5.31



Observing the two motions together

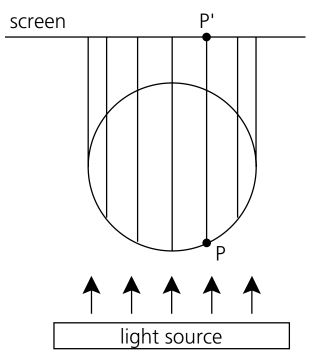
If you could set a pendulum swinging above an object moving in a horizontal circle at constant speed, you could get the two moving side-by-side if the speeds were right (Figure 5.31).

If a light was used to project an image of the oscillating objects on to a wall, the shadows of the two objects would move in exactly the same manner (Figure 5.32).

Consider point P making a complete revolution of the circle in Figure 5.32. The point P′ makes a complete oscillation on the straight line of the pendulum. Equally spaced points on the circle project as points on the line as shown. This illustrates that maximum acceleration occurs at the maximum amplitude of the pendulum, and minimum acceleration occurs when the amplitude is a minimum. This is a characteristic of SHM.

In more formal language: Simple harmonic motion is the projection of uniform circular motion on the diameter of the circle in which the circular motion occurs.

Figure 5.32



Practice questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: * = low; ** = medium; *** = high.

Review — applying principles and problem solving

***23** A boy sitting in a train carriage moving at constant velocity throws a ball straight up in the air.
- **(a)** Will the ball fall behind him, in front of him or into his hands?
- **(b)** What happens if the train accelerates while the ball is in the air?
- **(c)** What happens if the train turns a corner while the ball is in the air?

***24** A motorcycle is driven off a cliff at a horizontal velocity of 15 m s^{-1} and takes 2.5 seconds to reach the ground below. Calculate **(a)** the height of the cliff; **(b)** the distance out from the base of the cliff that the motorcycle lands; **(c)** the impact velocity.

***25** When a wedding ring is thrown horizontally out of a fifth floor window 15 m off the ground, it lands 7.5 m out from the base of the building. Calculate **(a)** the throwing speed; **(b)** the impact velocity; **(c)** how long the marriage will last.

***26** A golf ball is hit by a club and moves off with a velocity of 30 m s^{-1} at an angle of 55° to the horizontal. Find the following:
- **(a)** The initial vertical and horizontal components of the velocity.
- **(b)** The maximum height reached.
- **(c)** The time of flight.
- **(d)** The horizontal range.

***27** A soccerball is kicked off the ground at an angle of 20° to the horizontal. It moves away at 30.0 m s^{-1}. Calculate **(a)** the vertical velocity after 0.5 s; **(b)** the velocity of the ball after 1.0 s; **(c)** the maximum height reached; **(d)** the time of flight; **(e)** the range of the ball.

****28** The world record for fresh hen's egg throwing is 96.90 m, set in 1981. Assuming no air resistance, what would have been the **(a)** throwing speed; **(b)** elevation angle; **(c)** maximum height; **(d)** time of flight?

***29** A car of mass 2250 kg is travelling around a circular track of radius 90 m at a constant speed of 30 m s^{-1}. Calculate **(a)** the centripetal acceleration; **(b)** the centripetal force; **(c)** what time it takes to complete one lap.

****30** An aviator, pulling out of a dive, follows the arc of a circle and is said to have experienced 3 '***g***'s. Explain what this means.

***31** In the Bohr model of a hydrogen atom, an electron orbits a proton in a circle of radius 5.28×10^{-11} m with a speed of 2.18×10^{6} m s^{-1}. What is the acceleration of the electron in this model?

***32** Convert the following:
- **(a)** 1 rad to degrees;
- **(b)** 8.5 rad to degrees;
- **(c)** 90° to rad;
- **(d)** 5 rpm to rad s^{-1};
- **(e)** 100 rad s^{-1} to rev per second;
- **(f)** 2 revolutions of a 50 cm radius circle to metres;
- **(g)** 20 rad s^{-1} of a 1.5 m radius circle to linear m s^{-1}.

***33** An amusement park Ferris wheel moves in a horizontal circle of 15 m radius and completes five turns every minute.
- **(a)** What is the acceleration of a passenger at **(i)** the highest point; **(ii)** the lowest point?
- **(b)** If the passenger has a mass of 65 kg, what would her apparent weight be at these two points?

****34** The maximum breaking strain of a piece of cord is 250 N. What is the maximum rpm at which the line can retain a 3 kg mass swung in a 1.8 m radius circle?

****35** A flywheel of radius 65 cm is rotating at 2000 rpm. Calculate **(a)** the angular velocity; **(b)** the linear velocity of a point on the rim.

***36** A light spring stretches by 20 cm when a mass of 200 g is hung vertically from it.
- **(a)** Calculate its spring constant.
- **(b)** When it is set oscillating, what would be its period?
- **(c)** What would be its frequency be?

***37** Determine the period of a pendulum with a length of 45.0 cm at a place where: **(a)** $\boldsymbol{g}$ = 9.805 m s^{-2}; **(b)** 9.785 m s^{-2}.

****38** When travelling upwards in a lift at constant speed a pendulum has a period of 1.30 s. When accelerating, however, the period becomes 1.22 s. Calculate **(a)** the length of the pendulum; **(b)** the acceleration of the lift. Assume $\boldsymbol{g}$ = 9.8 m s^{-2}.

****39** A centripetal force experiment was conducted to find relationships between some of the variables.

Part A was conducted to determine the relationship between centripetal force ($\boldsymbol{F}_c$) and velocity ($\boldsymbol{v}$) in horizontal circular motion. Using the experimental set-up as shown in Figure 5.33, a rubber stopper was swung at constant speed in a horizontal circle. The hanging mass, which provided the centripetal force, was varied and the time for 10 complete revolutions of the rubber stopper was noted. In all cases the radius of revolution (r) was kept at 1.5 m and the same rubber stopper was used each time. The mass of the rubber stopper (m_s) was 50 g.

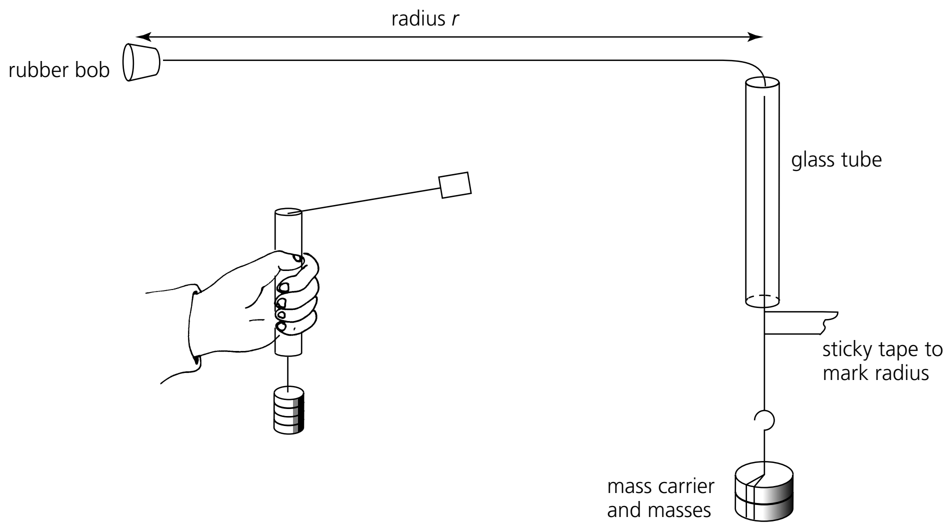


Figure 5.33
For question 39.

The results shown in Table 5.3 were obtained.

Table 5.3 CENTRIPETAL FORCE DATA (PART A)

m_h(g)	m_s(g)	RADIUS (m)	TIME FOR 10 REVOLUTIONS (s)
50	50	1.5	24.6
100	50	1.5	17.4
150	50	1.5	14.2
200	50	1.5	12.3
250	50	1.5	11.0

(a) Calculate the centripetal force ($\boldsymbol{F}_c$) provided by the hanging mass for each stage.
(b) Calculate the period (T) and the linear velocity ($\boldsymbol{v}$) of the rubber stopper for each stage.
(c) Plot $\boldsymbol{F}_c$ vs $\boldsymbol{v}$ using the x-axis for $\boldsymbol{v}$.
(d) Suggest a possible relationship between $\boldsymbol{F}_c$ and $\boldsymbol{v}$. Plot the appropriate data to confirm or refute the suggested relationship. Does it agree with the centripetal force formula?

Part B Relationship between radius and velocity. The above experiment was repeated with a 100 g rubber stopper. This time the hanging mass was kept constant at 100 g while the radius of revolution was varied. Again, the time for 10 revolutions was measured and the data recorded in Table 5.4.

Table 5.4 CENTRIPETAL FORCE DATA (PART B)

m_h(g)	m_s(g)	RADIUS (m)	TIME FOR 10 REVOLUTIONS (s)
100	100	0.8	17.9
100	100	1.0	20.1
100	100	1.2	22.0
100	100	1.5	24.8

(e) Calculate and plot r vs $\boldsymbol{v}$ using $\boldsymbol{v}$ for the x-axis again.
(f) Suggest a relationship and plot to confirm.
(g) Does it agree with the formula?
(h) What would the shape of an $\boldsymbol{F}_c$ vs r graph look like (r on the x-axis) if m_s and $\boldsymbol{v}$ were kept constant?

**40 An experiment was carried out to establish the relationship between length and period of a simple pendulum. A brass bob was tied to a length of cotton thread and as its length was increased, the time for 10 oscillations was noted. The results are as follows:

Length (cm)	20.0	25.0	35.0	40.0	45.0
Time for 10 swings (s)	9.0	10.0	11.9	12.7	13.6

(a) Plot a graph to establish the possible relationship.
(b) Plot another graph to confirm the suggested relationship.
(c) From either graph, determine the time for 10 swings if the length was **(i)** 30.0 cm; **(ii)** 60 cm.

**41 The following data (Table 5.5) were taken from *Overlander 4WD* magazine's road test of some four wheel drives.

Table 5.5 FOUR WHEEL DRIVE ENGINE DATA

	LANDCRUISER	LAND ROVER	PAJERO	NISSAN PATROL
Capacity	4.477 L	3.528 L	2.972 L	4.169 L
Maximum power	158 kW at 4600 rpm	114 kW at 4700 rpm	109 kW at 5000 rpm	129 kW at 4000 rpm
Maximum torque	373 Nm at 3200 rpm	271 Nm at 3000 rpm	234 Nm at 4000 rpm	330 Nm at 3200 rpm

Comment critically on the following assertions by referring to the data.
(a) The bigger the engine capacity the greater the power and torque.
(b) Smaller capacity engines have to rev at a higher rate (rpm) for their maximum power and torque than do bigger engines.
(c) Engines have to turn at a higher rpm to get maximum power than they have to for maximum torque.

Extension — complex, challenging and novel

***42 A dart is thrown horizontally towards a bull's eye of a dart board but it strikes the 3 on the bottom of the board directly underneath, 0.19 s later (Figure 5.34). What is the distance from the bull's eye to the 3?

***43 An arrow is fired off a 50 m cliff at an angle of 20° above the horizontal. If it has an initial velocity of 35 m s^{-1} and strikes the rocks below, calculate **(a)** the time of flight; **(b)** the impact velocity; **(c)** how far out from the base of the cliff the arrow strikes the ground.

***44 In the 1968 Olympics in Mexico City, Bob Beamon shattered the world long jump record with a jump of 8.90 m. His speed on take-off was measured at 9.5 m s^{-1}, about equal to that of a sprinter. How close did he come to achieving maximum range for this speed in the absence of air resistance? The value of $\boldsymbol{g}$ in Mexico City is 9.78 m s^{-2}.

***45 A plane, diving at an angle of 53.0° to the vertical, releases a projectile at an altitude of 730 m. The projectile hits the ground 4.50 s after being released (Figure 5.35).
(a) What is the speed of the plane?
(b) How far did the projectile travel horizontally during its flight?
(c) What is the impact velocity?

Figure 5.34
For question 42.

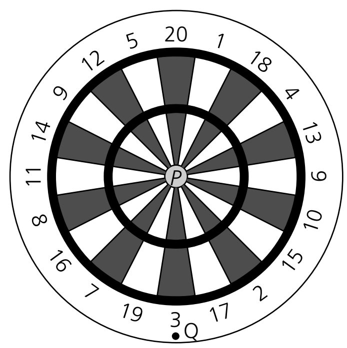
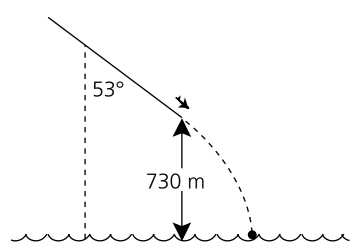


Figure 5.35
For question 45.



*****46** A person stands against the vertical walls of a cylindrical rotor in an amusement park. As it rotates, she feels pressed against the walls of the rotor and she remains suspended there as the floor moves away. The centripetal force is the normal force with which the wall pushes on the person.

(a) If the rotor has a radius of 2.1 m and the coefficient of friction between the person and the wall is 0.40, calculate the minimum speed of the rotor to just keep the person suspended on the wall.

(b) If the person has a mass of 49 kg, calculate the centripetal force acting on her.

*****47** A pilot of mass 80 kg who has been diving his plane vertically downwards with a velocity of 120 m s^{-1} pulls out of his dive by changing his course to a circular path of radius 800 m. If he maintains his constant speed,

(a) what will be his maximum acceleration;

(b) if he can stand 4.5 ***g*** without blacking out, will he remain conscious;

(c) what is the maximum force that his seat exerts on him?

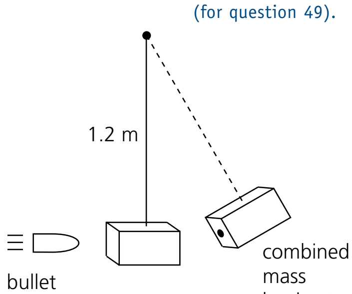
*****48** For a simple pendulum undergoing four oscillations:

(a) Draw graphs showing the relationships between the following variables **(i)** displacement vs time; **(ii)** velocity vs time; **(iii)** acceleration vs time; **(iv)** velocity (*y*-axis) vs displacement (*x*-axis). Remember that ***s***, ***v*** and ***a*** are vector quantities so have + and – direction.

(b) Repeat the question above but this time imagine that the pendulum is 'damped', that is, friction causes it to slow down as it moves.

(c) The ***v*** vs ***s*** graph for damped motion is said to be a 'strange attractor'. Look up a book on chaos theory to find out what this means.

Figure 5.36
The ballistic pendulum (for question 49).



*****49** A bullet of mass 10.0 g is fired into a 'ballistic pendulum' — a wooden block, which has a mass of 1.000 kg. The wooden block is suspended from a string 1.20 m long as shown in Figure 5.36. The bullet enters the stationary block and remains embedded in it. Using the value of 9.80 m s^{-2} for ***g***, calculate the period of the pendulum.

*****50** *Courier-Mail* correspondent Dave Barry wrote about an exciting new sport taking off in Florida, USA. It's called 'car bowling' where you go up in an airplane and drop bowling balls on cars. He wrote: 'Women think — "You drop what, on what, from what?" whereas men think "When can I do this?" You fly over an old car on a private runway at 145 km/h at an altitude of 20 m and attempt to hit the car with a bowling ball. The beauty of car bowling is that even if you miss, you get to watch a bowling ball bounce along a runway. It's amazing.'

(a) How far horizontally should you be from the car when you drop the ball?

(b) What would your 'sight angle' be at this point? (Sight angle is the angle between the line to the target and the vertical at the drop point) **(c)** Assume that the impact angle on contact with the runway equals the launch angle after contact, but with a 20% reduction in speed. Calculate **(i)** the maximum height and **(ii)** the distance the ball travels before its next impact.

TEST YOUR UNDERSTANDING

(Answer true or false)

- The period of a pendulum depends on the amplitude.
- A pendulum accelerates through the lowest part of its swing.
- An object moving in a circle with constant speed has no acceleration.